Trigonometry 2

1. Solve $\sin\theta \csc\frac{\theta}{2} - \cot\frac{\theta}{2} = 3\left(1 - 2\sin\frac{\theta}{2}\right)$ for $-360° \leq \theta \leq 360°$.

2. Show that for all values of θ ,

(a) $cos\ \theta + cos\ \left(\theta + \frac{2}{3}\pi\right) + cos\ \left(\theta + \frac{4}{3}\pi\right) = 0$

(b) $sin^2\ \theta + sin^2\ \left(\theta + \frac{2}{3}\pi\right) + sin^2\ \left(\theta + \frac{4}{3}\pi\right) = \frac{3}{2}$

(c) $cos^3\ \theta + cos^3\ \left(\theta + \frac{2}{3}\pi\right) + cos^3\ \left(\theta + \frac{4}{3}\pi\right) = \frac{3}{4}\cos 3\theta$.

3. Prove $-\frac{1}{\sec 2x} \equiv \frac{2(sin^3\ x - cos^3\ x)}{\sin x + \cos x} + \frac{\cos 2x}{(\sin x + \cos x)^2}$.

4. Given that $y = \frac{5+4\cos\theta}{1+\sin\theta}$. Use the substitution $t = \tan\frac{\theta}{2}$,show that $y = \frac{9+t^2}{1+2t+t^2}$.

Hence, or otherwise, prove that $y \geq \frac{9}{10}$.

5. Sketch a graph $y = \cos 2\theta$ in the range $0 \leq \theta \leq \pi$. Hence, find the set of values of θ, where $0 \leq \theta \leq \pi$, satisfying the inequality $4\sin^2\theta \geq 2 - \sqrt{3}$.

6. Solve $\sin 2\theta - 2\sin\theta = 1$ for $\theta = 0°\ to\ 360°$.

7. If $f(r) = \cos 2r\theta$, simplify $f(r) - f(r-1)$. Use your result to find the sum of the first n terms of the series $\sin 3\theta + \sin 5\theta + \sin 7\theta + \cdots$.

8. Solve the equation $cos^{-1}2x + sin^{-1}x = \frac{\pi}{3}$.

9. Prove that $\frac{2\sin 4\theta - \sin 6\theta - \sin 2\theta}{2\sin 4\theta + \sin 6\theta + \sin 2\theta} = \tan^2\theta$.

Hence, find the value of $\tan^2 15°$, leaving your answer in surd form.